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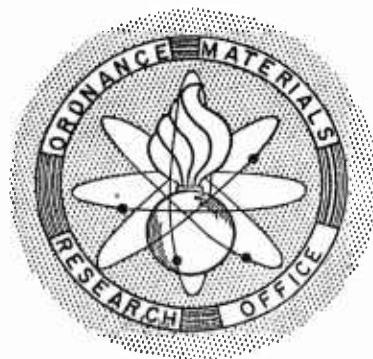
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HARTREE-FOCK WAVE FUNCTIONS FOR THE 4p SHELL ATOMS

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HARTREE-FOCK WAVE FUNCTIONS FOR THE 4p SHELL ATOMS

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ABSTRACT

Conventional (or restricted) Hartree-Fock wave functions have been obtained for Ga<sup>+</sup>, Ge<sup>++</sup>, Br<sup>-</sup>, Rb<sup>+</sup> and the neutral atoms Zn, Ga, Ge, As, Se, Br and Kr. Analytic Hartree-Fock methods utilizing a version of Nesbet's symmetry and equivalence restrictions, were used to obtain the solutions in analytic form (sums of powers of r times exponentials). Results are given in the form of wave functions, one-electron energies, and total energies. Comparison is made with the earlier results for Zn and Kr, the only atoms in this group for which H-F solutions exist.

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## I. INTRODUCTION

It has become increasingly apparent that Hartree-Fock wave functions for free atoms and ions can play a vital role in providing approximate descriptions of molecular and solid state phenomenon. A striking example of this, and one which was quite unexpected, is the successful use<sup>(1)</sup> of free ion Hartree-Fock wave functions to describe the recent data on isomeric shifts (i.e., total s electron density at the nucleus) which have been determined by Mössbauer experiments. It goes without saying that basic to any such applications is not only the existence of such functions but their availability in a form suitable for computation. In some of our own studies involving the 4p atoms the complete lack of H-F wave functions, for this part of the periodic table, was only too obvious. In order to overcome this difficulty, we have determined Hartree-Fock solutions for Ga<sup>+</sup>, Ge<sup>++</sup>, Br<sup>-</sup>, Rb<sup>+</sup> and the neutral atoms Zn, Ga, Ge, As, Se, Br and Kr. Of these, Hartree-Fock (H-F) results have been previously available for only Zn<sup>(2)</sup> and Kr<sup>(3)</sup>. These are conventional or restricted Hartree-Fock solutions in that one-electron functions of the same shell are constrained to have the same radial dependence.<sup>(4)</sup>

As we have said, aside from their own inherent interest as a description of the electronic structure of free atoms, a major purpose of such calculations is to supply a starting point for further investigations. The spherical ion results have been used to obtain the Sternheimer quadrupole antishielding factors<sup>(5)</sup> ( $\gamma_{\infty}$ 's) as these extend our knowledge of theoretical values of  $\gamma_{\infty}$  to ions for which only estimates could be made previously<sup>(6)</sup> (except for Rb<sup>+</sup> for which Hartree calculations exist.) The H-F results have been utilized in several other investigations. Atomic scattering factors have been obtained<sup>(7)</sup> and the Ge results have supplied a starting point in an effort to improve on core and valence electron self-consistency in orthogonalized plane wave calculations<sup>(8)</sup> for germanium.

Analytic H-F methods,<sup>(9)</sup> utilizing a version of Nesbet's symmetry and equivalence restrictions,<sup>(10)</sup> have been used to obtain the wave functions. Some details of the method are discussed in Section II. A fuller discussion has been given previously<sup>(11)</sup> where H-F results for 3p shell atoms were reported. The H-F results appear in Section III along with comparisons of the previously available Zn and Kr functions.

## II. DESCRIPTION OF THE CALCULATION

Six of the eleven atoms and ions, for which H-F results will be reported, are closed shell ions. These can be straightforwardly handled<sup>(12)</sup> but the remaining five ions with their unfilled 4p and filled 2p and 3p shells raise a problem of maintaining orthogonality between p orbitals. We have discussed this matter at length when reporting<sup>(11)</sup> 3p atom solutions and the reader is referred there for a discussion of this. We have utilized the same version of Nesbet's symmetry and equivalence restrictions<sup>(10)</sup> as was used in the 3p atom investigation.<sup>(11)</sup> This procedure does not yield the best possible H-F results but it has been our experience,<sup>(13)</sup> and that<sup>(14)</sup> of investigators who have used the numerical H-F method, that the H-F eigenfunctions are insensitive to the way in which the orthogonality "problem" is treated. Because of this, application of the symmetry and equivalence restrictions yields results which are only negligibly inferior to the "best results." The discrepancies introduced are far smaller than the difference between the Hartree-Fock and the "true" many-electron wave functions. It is our opinion that if one requires superior many-electron eigenfunctions or eigenvalues one should be prepared to obtain these by going beyond the conventional Hartree-Fock formalism.

As already noted, we have utilized the analytic Hartree-Fock method to obtain our results. This method uses standard matrix techniques to obtain

orthonormal analytic Hartree-Fock radial orbitals,  $U_1(r)$ 's, of the form:

$$U_1(r) = \sum_j C_{1j} R_j(r) \quad (1)$$

Their normalization condition is:

$$\int_0^{\infty} |U_1(r)|^2 dr = 1 \quad (2)$$

and the basis functions  $R_j$ 's, are of the form:

$$R_j(r) = N_j r^{(\ell + A_j + 1)} e^{-Z_j r} \quad (3)$$

where  $\ell$  is the one-electron angular momentum quantum number appropriate for the one-electron orbital of which  $U_1(r)$  is the radial part. The  $N_j$  is a normalization constant and is expressible in terms of the other parameters, i.e.:

$$N_j = \left( \frac{(2Z_j)^{2\ell + 2A_j + 3}}{(2\ell + 2A_j + 2)!} \right)^{1/2} \quad (4)$$

$U_1(r)$ 's of common  $\ell$  value are constructed from a common set of  $R_j(r)$ 's. Given the basis sets, i.e. the  $R_j(r)$ 's, the problem is reduced to solving the Hartree-Fock integro-differential equations for the eigenvectors (the  $C_{1j}$ 's) and their eigenvalues. This is done by straightforward matrix diagonalization and manipulation and avoids the problems of numerical accuracy inherent in the integrations of the numerical H-F method.

Recently<sup>(15, 16, 17)</sup> several workers have overcome the problem of numerical integration accuracy. In particular Mayers<sup>(16)</sup> has not only obtained highly accurate numerical H-F solutions but he has also accurately evaluated (a not easy task!) the one and two-electron integrals making up the total energy. This will allow detailed comparisons to be made for the first time between numerical and analytic results.

In general the two approaches are complementary, sometimes one and sometimes the other provides a) the more suitable method of computation and/or b) results in a more convenient form. We have found the analytic approach to be very useful<sup>(17)</sup> when going beyond the conventional free atom H-F formalism. We have also found it convenient to have the resultant wave functions in analytic form.

In using the analytic approach we have replaced the problem of accuracy of numerical integration by the problem of choosing adequate basis sets. Let us consider this matter now. First there is the question of the size of the set. A small set is desirable because of economy in computer time and retains the advantages of wave functions of analytic form. These advantages come from the ease, accuracy, and convenience with which matrix elements can be obtained if the functions are in analytic form. Large basis sets allow greater accuracy of solution (providing that we maintain sufficient linear independence among the basis set, otherwise errors accumulate during matrix diagonalization). Having made the choice of the size of the basis set there is then the problem of choosing the individual  $R_j$ 's. In the present work we have relied heavily on earlier<sup>(2,18)</sup> H-F investigations for the iron series ions and so we will review these briefly. The first investigation<sup>(18)</sup> involved obtaining H-F solutions for iron series ions in  $3d^n$  (i.e. no 4s electrons) configurations. The basis sets were the largest that could be fitted onto the computer that was used.<sup>(18)</sup> Their size fixed, they were obtained by an extensive series of computations where the basis functions were varied. This search concentrated on obtaining good basis functions for V through Ni. This investigation was followed by a series of H-F calculations for the neutral  $3d^{n-2} 4s^2$  iron series atoms. These were done on a computer of larger capacity and the basis sets were obtained by adding additional s-like  $R_j$ 's (for the construction of the outer loop of the 4s orbitals) to the otherwise unmodified basis sets of the first investigation. An extensive basis function search was thus avoided at the

cost of building in the shortcomings of the first effort. Subsequently more accurate analytic H-F calculations were done for Zn ( $3d^{10}4s^2$ ,  $^1S$ ) which is reported here and for Fe ( $3d^64s^2$ ,  $^5D$ ), Mn $^{++}$  ( $3d^5$ ,  $^6S$ ) and Cu $^+$  ( $3d^{10}$ ,  $^1S$ ) which appear elsewhere.<sup>(20)</sup> The greatest discrepancies between calculations for the same state occur for Cu $^+$  and Zn which lie outside the group emphasized in the first basis function search.

The present computations were done on the I.B.M. 704 computer at Avco whose capacity (32,000 words) allowed the use of larger basis sets and we have taken advantage of this capability. In fact, the choice of  $R_j$ 's is based on the iron series  $R_j$ 's and on an extensive basis function search for Kr followed by tests on the resultant sets for other atoms. As a result the H-F functions appearing in this paper are more accurate than the earlier<sup>(2,18)</sup> iron series solutions. The parameters defining the  $R_j$ 's appear in Table I. We have reported (and used) screening constants ( $Z_j$ 's) with four digits after the decimal point. This does not mean that  $Z_j$ 's were uniquely established to this many digits. The investigations varying the  $Z_j$ 's carried this many digits and since these were kept in the final calculations they are reported here.

### III. RESULTS

The eigenvectors ( $C_{ij}$ 's) which define the  $U_j(R)$ 's in terms of the  $R_j(r)$ 's appear in Table II. Note that the  $C_{ij}$ 's are given for normalized  $R_j(r)$ 's. The  $C_{ij}$ 's have not been uniquely established to the number of digits quoted but with these digits they provide well-orthonormalized, well-defined Hartree-Fock orbitals. Total energies, one-electron energies ( $\epsilon_i$ 's), one-electron kinetic plus nuclear potential energies ( $K_i$ 's), and two-electron  $F^k(4p,4p)$  integrals<sup>(21)</sup> are listed in Table III. The quantities appearing there are accurately evaluated for the functions defined in Table II. The Hartree-Fock one-electron and total energies have not been rigorously

established to the number of digits quoted. More accurate solution of the Hartree-Fock equations (e.g. through the use of larger improved basis sets) will affect the last one or two-digits of these quantities. Note that Hartree atomic units have been used (1 au = 2 Ry).

The  $K_1$ 's which do not include interelectronic terms, offer a sensitive test of wave function behavior. For fixed Z, a contraction in a (r) causes an increase in the magnitude of its  $K_1$ . Inspection of the  $K_1$ 's of Table III shows in general that the  $U_1$ 's of any one element contract when going to a state of higher positive ionization. This is, of course, not surprising. Of greater interest is the exception to this rule which occurs for the 2p and 3p orbitals on going from a state with one or more 4p orbitals (such as Ga or Ge) to a state with none, (such as  $\text{Ga}^+$  or  $\text{Ge}^{++}$ ). The requirements of obtaining three simultaneous (hence orthogonal) p-like eigenfunctions for the former state and the partial relaxation of this for the latter has caused a reverse trend in  $K_{2p}$  and  $K_{3p}$ . It should be noted that the resultant 2p and 3p orbital variations are small.

Let us now compare the present results with those previously available for Kr and Zn. Worsley's numerically obtained<sup>(8)</sup>  $\epsilon_1$ 's for Kr appear in Table IV.  $K_1$ 's and the total energy were not obtained by her. We see that there is substantial agreement between the two sets of results. More detailed comparisons can be made with the earlier obtained<sup>(2)</sup> analytic Zn results where  $K_1$ 's,  $\epsilon_1$ 's, and the total energy are available. These appear in Table V. Comparison of  $\epsilon_1$ 's show larger discrepancies than were seen for Kr. This is due to the previously discussed deficiencies in the basis set of the earlier calculation. If one chose to inspect the  $\epsilon_1$ 's to learn of orbital variation as has often been done (out of necessity) in the past, one would conclude that the 4s orbitals have changed least (the  $\epsilon_{4s}$ 's being in best agreement between calculations) and that the 3d orbital of the present calculation is the more contracted (since its  $\epsilon_{3d}$  is more negative). On the other

hand inspection of the  $K_1$ 's shows that neither conclusion is correct. The 4s orbitals show the greatest, not the least, modification and the present calculation's 3d orbital is expanded and not contracted relative to the earlier results. In addition to inspecting individual  $\epsilon_1$ 's, sums<sup>(22)</sup> of  $\epsilon_1$ 's (which also appear in Table V) have in the past been used to estimate the variation in total energy. Such sums suggest that there has been a 1 au change in total energy when in reality a change of one fiftieth that size has occurred. In other words the one-electron energies supply an often misleading "yardstick" to wave function behavior.

#### IV. ACKNOWLEDGEMENT

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- 3B. H. Worsley, Proc. Roy. Soc. A247, 390 (1958).
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- 6R. M. Sternheimer and H. M. Foley, Phys. Rev. 102, 731 (1956) and E. G. Wikner and T. P. Das, Phys. Rev. 109, 360 (1958).
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- 9C. A. Coulson [Proc. Cambridge Phil. Soc. 34, 204 (1938)] appears to have been the first to have used an expansion technique in a molecular problem, while C. C. J. Roothaan [Revs. Modern Phys. 23, 89 (1951)] presented the approach in a particularly desirable form for closed-shell molecules. Nesbet, with his symmetry and equivalence restrictions, extended the method to nonclosed shells and emphasized its use for atomic cases [see reference 10 and also Quarterly Progress Reports No. 15, January, 1955, p. 10; No. 18, April, 1955, p. 38 and p. 41; No. 18, October, 1955, p. 4, Solid-State and Molecular Theory Group, Massachusetts Institute of Technology, Cambridge, Massachusetts (unpublished)]. Recently C. C. J. Roothaan [Revs. Modern Phys., 32, 179 (1960)] has extended his formalism to cover the nonclosed shell case for the conventional restricted Hartree-Fock method where nonzero off-diagonal Lagrange multipliers occur. S. Huzinaga (Phys. Rev. 120, 866 (1960); *ibid* 122, 131 (1961)) has extended Roothaan's formalism further.
- 10R. K. Nesbet, Proc. Roy. Soc. A230, 312 (1955).
- 11R. E. Watson and A. J. Freeman, Phys. Rev. (to be published); Section II and the appendix of this paper supply sufficient information for the reader to construct the Hartree-Fock equations solved in the present paper.
- 12See D. R. Hartree, The Calculation of Atomic Structures, (John Wiley & Sons Inc., New York, 1957) for details concerning the conventional Hartree-Fock formalism and for the derivation of Hartree-Fock equations.

- <sup>13</sup>This observation is based on the work reported in ref. 11 and in R. K. Nesbet and R. E. Watson, Ann. Phys. (New York) 9, 280 (1960).
- <sup>14</sup>In two cases (D. R. Hartree and W. Hartree, Proc. Roy. Soc. A193, 299 (1948); W. Hartree, D. R. Hartree and M. Manning, Phys. Rev. 60, 857 (1948)) small "off-diagonal" Lagrange multipliers were included for orthogonality, otherwise [D. R. Hartree and W. Hartree, Proc. Roy. Soc. A164, 167 (1938); D. A. Goodings (to be published); and unpublished work of D. F. Mayers (1958) the problem was ignored, i.e., the multipliers were set equal to zero and orthogonality was reasonably maintained.
- <sup>15</sup>D. F. Mayers (to be published) has obtained, among others, numerical Hartree-Fock results for Zn but at the time of writing these results were in a preliminary stage and thus were not available for comparison.
- <sup>16</sup>D. A. Goodings (to be published) has obtained accurate numerical "unrestricted" Hartree-Fock solutions for a number of low Z atoms.
- <sup>17</sup>For example, the analytic method was used to obtain Hartree-Fock solutions (1) for ions in external "crystalline" fields [R. E. Watson, Phys. Rev. 117, 742 (1960) and R. E. Watson and A. J. Freeman Phys. Rev. 120, 1134 (1960)]; (2) for a non-magnetic ion in the "exchange field" of neighboring magnetic ions [A. J. Freeman and R. E. Watson, Bull. Am. Phys. Soc. 6, 234 (1961)]; and (3) to obtain one-electron orbitals which are not separable into a product of a radial and an angular function C. Sonnenschein, (to be published). In (2) and (3) the analytic method has several advantages in making the computations possible.
- <sup>18</sup>R. E. Watson, Phys. Rev. 119, 1934 (1960).
- <sup>19</sup>M.I.T.'s Whirlwind I was used.
- <sup>20</sup>For definitions see E. U. Condon and G. H. Shortley, The Theory of Atomic Spectra, (Cambridge University Press, Cambridge, 1953), p. 177.
- <sup>21</sup>The Mn<sup>++</sup> results appear in an investigation [R. E. Watson and A. J. Freeman (to be published)] of hyperfine effects, those for Cu<sup>+</sup> in an investigation of quadrupole polarizabilities [A. J. Freeman and R. E. Watson, Bull. Am. Phys. Soc. 6, 166 (1961)], and the Fe results are unpublished.

Table I Parameters  $\{t_j\}$  and  $z_j$ , which define the basis orbitals  $(R_j \mathbf{l}_j)$

$\mathbf{l}_j$ used for the construction of		$j$	$t_j$	$Zn(\mathbf{l}_j \mathbf{s}, \mathbf{l}_j)$	$S(\mathbf{l}_j \mathbf{s}, \mathbf{l}_j)$	$\beta_e(\mathbf{l}_j \mathbf{s}, \mathbf{l}_j)$	$A_S(\mathbf{l}_j \mathbf{s}, \mathbf{l}_j, \mathbf{4}_S)$	$S(\mathbf{4}_S \mathbf{2}_{4p}, \mathbf{2}_P)$	$B(\mathbf{4}_S \mathbf{2}_{4p}, \mathbf{5}, \mathbf{2}_P)$	$B(\mathbf{4}_S \mathbf{2}_{4p}, \mathbf{6}, \mathbf{1}_S)$	$Kr(\mathbf{4}_S \mathbf{2}_{4p}, \mathbf{6}, \mathbf{1}_S)$	$Rb(\mathbf{4}_S \mathbf{2}_{4p}, \mathbf{6}, \mathbf{1}_S)$	$Ga(\mathbf{4}_S \mathbf{2}_{4p}, \mathbf{6}, \mathbf{1}_S)$	$Ga(\mathbf{4}_S \mathbf{2}_{4p}, \mathbf{2}_P)$	
$\mathbf{p}$ orbitals ( $\ell = 1$ )		1	0	31.6657	32.7951	33.9305	35.0739	36.2133	37.3527	38.4921	39.6315	32.7951	32.9345	32.9345	
		2	1	27.9415	28.9818	30.0221	31.0224	32.1027	33.1430	34.1030	35.2235	36.2235	28.9818	30.0221	30.0221
		3	1	14.4013	14.5772	15.5534	16.1290	16.7049	17.2808	17.2808	18.4325	19.4325	14.9772	15.5534	15.5534
		4	2	13.6516	14.1895	14.7773	15.2651	15.8029	16.3107	16.3407	16.8785	17.4663	14.1895	14.7773	14.7773
		5	2	6.38235	7.4930	7.4622	7.7134	8.1005	8.4198	8.4198	8.7390	9.0582	7.1139	7.4622	7.4622
		6	2	5.1970	5.4823	5.7676	6.0279	6.3382	6.6235	6.6235	6.9088	7.1941	5.4823	5.7676	5.7676
		7	3	5.4426	5.7224	5.9822	6.2020	6.5018	6.7616	6.7616	7.0214	7.2812	5.7224	5.9822	5.9822
		8	3	2.3894	2.6961	3.0892	3.1159	3.3714	3.6569	3.6569	3.8824	4.1379	2.6049	2.8604	2.8604
		9	3	1.6592	2.0185	2.1084	2.1925	2.2903	2.4802	2.4802	2.5561	2.6561	1.507	1.6366	1.6366
		10	3	0.8829	1.0954	1.3508	1.3553	1.4461	1.5869	1.5869	1.7277	1.9464	1.0237	1.1645	1.1645
$\mathbf{d}$ orbitals ( $\ell = 2$ )		1	0	19.6537	20.3930	21.1223	21.8516	22.5809	23.3102	23.3102	24.0396	24.7689	20.3930	24.4223	24.4223
		2	0	12.3337	12.9975	13.6615	14.3254	14.9833	15.5532	15.5532	16.3171	16.3810	12.3337	13.6615	13.6615
		3	1	11.7281	12.3476	12.9671	13.5666	14.2061	14.8256	14.8256	15.4451	16.0546	11.3176	12.3476	12.3476
		4	1	7.4809	7.9029	8.3219	8.7159	9.1669	9.5909	9.5909	10.0129	10.4346	7.9029	8.3219	8.3219
		5	1	4.6219	4.9255	5.2281	5.5227	5.8338	6.1395	6.1395	6.4435	6.7171	4.9255	5.2281	5.2281
		6	2	4.3975	4.5067	4.7359	4.9551	5.1913	5.4235	5.4235	5.6527	5.8819	4.5067	4.7359	4.7359
		7	2	1.9142	2.0473	2.2704	2.4835	2.7165	2.9397	2.9397	3.1675	3.3859	2.0473	2.2704	2.2704
		8	2	1.0264	1.0911	1.2458	1.4005	1.5552	1.7099	1.7099	1.8646	2.0016	1.0911	1.2458	1.2458
		9	2	0.6758	0.7355	0.8512	0.9886	1.0866	1.2043	1.2043	1.3220	1.4893	0.7355	0.8512	0.8512
$\mathbf{f}$ orbitals ( $\ell = 3$ )		1	0	1.8017	2.0478	2.2939	2.5400	2.7861	3.0322	3.0322	3.4679	3.2783	2.0478	2.2939	2.2939
		2	0	2.9851	3.3364	3.6377	4.0390	4.3903	4.7416	4.7416	5.0929	5.4442	3.3364	3.6377	3.6377
		3	0	5.3914	5.7709	6.1504	6.5299	6.9094	7.2889	7.2889	7.6514	8.0179	5.3914	5.7709	5.7709
		4	0	7.9320	8.3527	8.7934	9.2244	9.6558	10.0855	10.0855	10.5162	10.9619	6.3527	6.934	6.934
		5	0	13.5738	14.0706	14.5674	15.0642	15.5610	16.0578	16.0578	16.5546	17.0514	14.5574	14.0706	14.0706

Table II The eigenvectors  $\{e_i\}$  defining the Hartree-Fock radial functions  $\{u_i\}$  in terms of the basis sets  $\{f_j\}$ 

$i = 1s$	$\text{R}(\text{4s}^2, \text{1s})$	$\text{E}(\text{4s}^2, \text{1s})$	$\text{G}(\text{4s}^2, \text{1s})$	$\text{As}(\text{4s}^2\text{4p}^5, \text{2p})$	$\text{Br}(\text{4s}^2\text{4p}^5, \text{2p})$	$\text{Br}(\text{4s}^2\text{4p}^5, \text{1s})$	$\text{Kr}(\text{4s}^2\text{4p}^5, \text{1s})$	$\text{Kr}(\text{4s}^2\text{4p}^5, \text{1s})$	$\text{Kr}(\text{4s}^2\text{4p}^5, \text{1s})$	$\text{Kr}(\text{4s}^2\text{4p}^5, \text{1s})$	$\text{Ge}(\text{4s}^2\text{4p}^5, \text{2p})$	
1	-0.927097498	.904136614	.90136614	.898777456	.89532411	.89190601	.89567625	.89193367	.89193367	.89193367	.89193367	
2	-0.8641141	.126262149	.12455705	.12810632	.13169827	.13455985	.1375253	.14017258	.12591008	.12448155	.12448155	
3	-0.1053316	.01260815	.01455509	.01587817	.01830664	.01717456	.02031053	.02161235	.01252562	.01432529	.01432529	
4	-0.0866203	.01652202	.01232253	.01373670	.01615916	.01667612	.01814546	.01938976	.01047243	.01253392	.01253392	
5	-0.01725706	.0315043	.03086332	.03191332	.01295477	.01624268	.01836205	.021256752	.05513860	.02213351	.02696688	
6	-0.02939256	.03637168	.04995466	.05650929	.06580494	.0650205	.07373764	.0737781	.03419191	.01345700	.01345700	
7	-0.01529476	.01980535	.02568854	.024955716	.03320697	.03103615	.0342123	.0342123	.01866719	.02180156	.02180156	
8	-0.00032494	.00601449	.01212153	.00104625	.00167549	.00168172	.00195735	.00195735	.00368684	.00850301	.00971652	
9	-0.00019724	.00038622	.00071965	.00082518	.00103568	.00103598	.001650279	.001650279	.001650279	.00048589	.00048589	
10	-0.00036165	.00044544	.00023725	.00023737	.00050934	.000506230	.00051705	.00051705	.00051705	.00012850	.00012850	
<hr/>												
1 = 2s	1	-0.77983383	-0.27991606	-0.27993746	-0.2799663	-0.28003250	-0.28007968	-0.28007952	-0.27985448	-0.27985448	-0.27985448	
2	2	-0.16533476	-0.1677265	-0.16779003	-0.16884484	-0.1696212	-0.1704316	-0.17057357	-0.17131052	-0.16776161	-0.16776161	
3	3	-0.67515985	-0.68811921	-0.69112272	-0.69314295	-0.69515680	-0.69815236	-0.69815236	-0.6309776	-0.63093150	-0.63093150	
4	4	-0.45229455	-0.46030181	-0.46805722	-0.47539660	-0.48329752	-0.48956336	-0.49378768	-0.49587938	-0.46033998	-0.46283767	
5	5	-1.49010477	-1.3825694	-1.2627758	-1.17076234	-1.1503374	-1.09452110	-0.9184984	-0.82865556	-0.69208442	-1.38282659	
6	6	-1.95673030	-1.9562943	-1.4212278	-1.2183749	-1.1564016	-0.78385777	-0.15737214	-0.03317976	-0.6892998	-1.4233889	
7	7	-0.95829443	-0.78628390	-0.62023650	-0.49387885	-0.45461675	-0.43862135	-0.03311355	-0.0338203	-0.7833900	-0.6377434	
8	8	-0.00051434	-0.000532612	-0.00051434	-0.000234	-0.00011532	-0.00011532	-0.02846438	-0.02546438	-0.02546438	-0.00244928	
9	9	-0.00035545	-0.00019249	-0.00027554	-0.00035554	-0.00030383	-0.00030383	-0.01195256	-0.01195256	-0.00022117	-0.00022117	
10	10	-0.00013818	-0.00005129	-0.00013323	-0.00016390	-0.00013592	-0.00013592	-0.00072679	-0.00072679	-0.00058393	-0.00058393	
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1 = 3s	1	-1.0598677	-1.07023238	-1.08138654	-1.0956351	-1.1040375	-1.115294	-1.1146347	-1.1256938	-1.1326858	-1.0702374	
2	2	-0.05959551	.05111038	.06272258	.06323238	.06509332	.06557339	.06655214	.06677341	.06677341	.06677341	
3	3	-0.29825317	-0.24971526	-0.25129423	-0.25040729	-0.25155304	-0.25155304	-0.23175763	-0.2519562	-0.2521847	-0.2496926	
4	4	-0.343566733	-0.3471174	-0.35081615	-0.35561995	-0.3661198	-0.37138991	-0.3932905	-0.37228707	-0.3832364	-0.35933748	
5	5	-0.73912135	-0.64432040	-0.551454568	-0.53802877	-0.49211037	-0.42565703	-0.416833164	-0.37398855	-0.32463808	-0.66737595	
6	6	-0.08520475	-0.10378904	-0.10378904	-0.28491788	-0.30413184	-0.33992537	-0.5100294	-0.60272688	-0.6896185	-0.05620145	
7	7	-0.53412626	-0.4804505	-0.33098663	-0.33098663	-0.28842826	-0.28842826	-0.1653722	-0.14274180	-0.14274180	-0.59838350	
8	8	-0.03048772	-0.03446115	-0.03591374	-0.03085439	-0.02959984	-0.02959984	-0.02480334	-0.03146733	-0.03261221	-0.03092930	
9	9	-0.1676536	-0.17934908	-0.16585759	-0.16443164	-0.12442338	-0.1012442338	-0.01167423	-0.01233601	-0.01656959	-0.01534694	
10	10	-0.0078473	-0.00555008	-0.00601189	-0.00546188	-0.00462019	-0.00462019	-0.00458575	-0.00428437	-0.00428437	-0.004646920	
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1 = 4s	1	-0.02116275	-0.02657713	-0.03089578	-0.03012110	-0.03230841	-0.03420874	-0.03420874	-0.03330359	-0.03589367	-0.02487901	
2	2	-0.01228875	-0.01503131	-0.01748491	-0.01700713	-0.01831023	-0.01947774	-0.01947774	-0.01846686	-0.02058588	-0.01409084	
3	3	-0.05101164	-0.05113794	-0.07020664	-0.05720889	-0.07126779	-0.07482571	-0.06874398	-0.0775195	-0.08238557	-0.05759839	
4	4	-0.06945675	-0.09334884	-0.1105925	-0.11036395	-0.12015399	-0.1283010	-0.12948274	-0.1352181	-0.14625994	-0.08700354	
5	5	-0.05269363	-0.05269363	-0.15760514	-0.20541818	-0.20283058	-0.22335614	-0.23155680	-0.23182777	-0.23657743	-0.17852280	
6	6	-0.22885257	-0.10691237	-0.06533125	-0.05249001	-0.04097520	-0.04726555	-0.04726555	-0.05233353	-0.08356018	-0.12854594	
7	7	-0.06865891	-0.13267354	-0.12317176	-0.1277892	-0.12015399	-0.12948274	-0.130277922	-0.12098864	-0.12098864	-0.12098864	
8	8	-0.40659294	-0.42317176	-0.37269587	-0.37269587	-0.34397419	-0.34397419	-0.34397419	-0.50265503	-0.52472702	-0.10279338	
9	9	-0.59138608	-0.67785939	-0.70581098	-0.61658579	-0.57986312	-0.57503411	-0.5738512	-0.5732077	-0.6184584	-0.47173308	
10	10	-0.12181579	-0.02950113	-0.02323804	-0.05953123	-0.07411869	-0.08039770	-0.11299232	-0.09000001	-0.04580513	-0.58289441	
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Table II The eigenvectors ( $C_i$ 's) defining the Hartree-Fock radial functions ( $U_i$ 's) in terms of the basis sets ( $R_i$ 's) (Continued)

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Table III Hartree-Fock one-electron energies ( $\epsilon$ , 1s), one-electron nuclear potential + kinetic energies ( $K_i$ , 1s), total energies and two-electron  $F_{ij}$ (3p, 3p) integrals. All energies are in atomic units ( $h\omega = 2 Ry$ ).

	$Z_0(4s^2, 4s)$	$Ga(4s^2, 1s)$	$Ga(4s^2, 1s)$	$Ss(4s^2, 4p, 1s)$	$Ss(4s^2, 4p, 1s)$	$Br(4s^2, 4p, 2p)$	$Br(4s^2, 4p, 2p)$	$Br(4s^2, 4p, 6, 1s)$	$Br(4s^2, 4p, 6, 1s)$	$Rb(4s^2, 4p, 6, 1s)$	$Rb(4s^2, 4p, 6, 1s)$	$Ga(4s^2, 4p, 2p)$	$Ga(4s^2, 4p, 2p)$
$\epsilon_{1s}$	-453.299	-279.120	-405.988	-432.578	-460.882	-480.052	-488.719	-503.167	-516.558	-532.811	-545.235		
$\epsilon_{2s}$	-48.458	-48.459	-52.887	-56.306	-60.666	-65.184	-64.858	-69.904	-75.250	-78.165	-82.145		
$\epsilon_{3s}$	-5.635	-6.697	-7.936	-8.029	-8.931	-9.869	-9.530	-10.850	-12.333	-6.394	-7.189		
$\epsilon_{4s}$	-0.294	-0.688	-1.175	-0.686	-0.837	-0.952	-0.683	-1.153	-1.721	-0.24	-0.553		
$\epsilon_{2p}$	-38.921	-12.795	-46.973	-50.150	-51.256	-58.516	-58.217	-63.011	-68.107	-42.490	-46.231		
$\epsilon_{3p}$	-3.837	-4.783	-5.893	-5.889	-6.061	-7.476	-7.438	-8.332	-9.688	-11.482	-5.060		
$\epsilon_{4p}$	-0.780	-1.494	-2.368	-2.112	-0.370	-0.403	-0.457	-0.137	-0.524	-1.009	-0.208		
$\epsilon_{3d}$	-	-	-	-	-2.649	-2.218	-2.880	-3.826	-4.933	-1.192	-1.633		
$K_{1s}$	-149.848	-460.346	-511.845	-544.343	-577.881	-621.339	-647.838	-684.338	-688.816	-700.346	-716.816		
$K_{2s}$	-110.571	-118.163	-126.007	-131.098	-142.441	-151.035	-159.033	-169.878	-168.973	-168.163	-170.015		
$K_{3s}$	-43.155	-46.384	-49.754	-53.241	-56.860	-60.602	-60.593	-64.057	-64.138	-66.381	-69.746		
$K_{4s}$	-12.216	-15.190	-17.748	-18.520	-20.566	-22.631	-22.216	-24.695	-27.447	-30.470	-34.534		
$K_{2p}$	-109.913	-117.508	-125.353	-133.449	-141.795	-150.391	-159.238	-168.336	-177.509	-175.354	-176.233		
$K_{3p}$	-40.981	-45.206	-47.584	-51.123	-58.518	-58.508	-62.386	-66.384	-74.233	-77.663	-80.056		
$K_{4p}$	-34.722	-38.541	-42.390	-46.140	-50.012	-51.976	-51.052	-53.954	-62.233	-38.505	-42.304		
$F_{1s}(4p, 4p)$	-36.05	-3942	-4293	-4643	-3975	-4643	-5229	-5650	-5229	-5650	-5652		
$F_{2}(4p, 4p)$	-1867	-2038	-2221	-2406	-2005	-2406	-2750	-3140	-2750	-3140	-3143		
Total Energy	-1977.841	-2074.529	-223.239	-2395.867	-2572.143	-2511.529	-2752.055	-2938.220	-2938.220	-2938.220	-2938.354		

Table IV Kr one-electron energies for the present calculation  
and as obtained by Worsley<sup>a</sup>

	Worsley's results <sup>a</sup>	present calculation
$\epsilon_{1s}$	-520. au	-520.167 au
$\epsilon_{2s}$	- 69.9	- 69.904
$\epsilon_{3s}$	- 10.85	- 10.850
$\epsilon_{4s}$	- 1.1515	- 1.153
$\epsilon_{2p}$	- 63.1	- 63.011
$\epsilon_{3p}$	- 8.335	- 8.332
$\epsilon_{4p}$	- 0.53	- 0.524
$\epsilon_{3d}$	- 3.80	- 3.826

<sup>a</sup>See ref. 3

Table V Zn one-electron energies ( $\epsilon$ 's), one-electron kinetic + nuclear potential energy integrals ( $K$ 's), sums of one-electron energies, and total energies for the present and earlier<sup>a</sup> analytic Hartree-Fock calculations.

	Earlier calculation <sup>a</sup>	Present calculation
$\epsilon_{1s}$	-353.261 au	-353.299 au
$\epsilon_{2s}$	-44.319	-44.358
$\epsilon_{3s}$	-5.600	-5.635
$\epsilon_{4s}$	-0.286	-0.291
$\epsilon_{2p}$	-38.882	-38.921
$\epsilon_{3p}$	-3.804	-3.837
$\epsilon_{3d}$	-0.751	-0.780
$K_{1s}$	-449.849	-449.848
$K_{2s}$	-110.569	-110.571
$K_{3s}$	-43.136	-43.155
$K_{4s}$	-12.060	-12.218
$K_{2p}$	-109.912	-109.913
$K_{3p}$	-40.996	-40.981
$K_{3d}$	-34.842	-34.722
Total Energy	-1777.823	-1777.843
$\sum_{i=1}^{30} \epsilon_i$	= -1070.558	-1071.514

<sup>a</sup>See ref. 2

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